Math 1190 B: Quiz 1 (2014 Fall)

(Time: 60 minutes, Thursday, October 16, 2014, 9:00-10:00 pm)

NAME (print): ____
(Family) (Given)

SIGNATURE: ____

STUDENT NUMBER: ______________________________________

Instructions:

1. Time allowed: 60 minutes.
2. Aids permitted: This is an open-book quiz. Textbooks and lecture notes are permitted.
3. Check that you have all 5 pages (including this title page).
4. Clearly indicate which part of the question you are answering in the space provided. If you need more space, be sure to indicate clearly where the rest of your answer is to be found.
5. Marks for each question are as indicated. You should allocate your time correspondingly.
6. Your work must justify the answer you give.
7. Sharing materials is not allowed.

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1	/10
2	/10
3	/15
4	/15

Question 1 (*10 marks*)**.** Determine whether this proposition is a tautology:

$$(p \wedge (p \to q)) \to q.$$

Justify your answer.

Solution 1. It is a tautology. (2 marks)

Indeed,

$$\begin{aligned}
(p \wedge (p \to q)) \to q &\equiv (p \wedge (\neg p \vee q)) \to q \\
&\equiv ((p \wedge \neg p) \vee (p \wedge q)) \to q \\
&\equiv (\mathbf{F} \vee (p \wedge q)) \to q \\
&\equiv (p \wedge q) \to q \\
&\equiv \neg(p \wedge q) \vee q \\
&\equiv (\neg p \vee \neg q) \vee q \\
&\equiv \neg p \vee (\neg q \vee q) \\
&\equiv \neg p \vee \mathbf{T} \\
&\equiv \mathbf{T}. & \text{(8 marks)}
\end{aligned}$$

□

Solution 2.

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(8 marks)

Thus $(p \wedge (p \to q)) \to q$ is a tautology. (2 marks) □

Question 2 (*10 marks*)**.** Let $\{a_n\}$ be a real sequence. The definition of "the limit of $\{a_n\}$ is L"

$$\lim_{n\to\infty} a_n = L$$

is expressed in English as:

- For every real number $\epsilon > 0$, there exists a positive integer K, such that for every positive integer $n > K$, it holds that $|a_n - L| < \epsilon$.

Use quantifiers and predicates to express the above statement.

Solution. $\forall\epsilon > 0\ \exists K\ \forall n((n > K) \rightarrow (|a_n - L| < \epsilon))$, (8 marks)
where the domain of ϵ consists of all real numbers, and the domain of K and n consists of all positive integers. (2 marks) □

Question 3 (*15 marks*)**.** Determine the negation of the English sentence

"Every student gets an A in some course"

in the following steps:

(a) Translate the statement into quantified statement.

(b) Determine the negation of the statement you get in (a), so that no negation is outside a quantifier.

(c) Translate the statement you get in (b) back into English.

Solution. (a) Denote by $P(x, y)$ the statement "The student x gets an A in the course y," where the domain of x consists of the students, and the domain of y consists of the courses. Then the original statement is

$$\forall x \exists y P(x, y). \qquad \text{(5 marks)}$$

(b) $\neg(\forall x \exists y P(x, y)) \equiv \exists x \neg(\exists y P(x, y)) \equiv \exists x \forall y \neg P(x, y)$. (5 marks)

(c) The answer is not unique. For example:

- There exists a student who does not get an A in any course.
- Some students do not get A in all the courses.
- Some student does not get an A in any course.
- There is at least one student who does not get an A in all the courses. (5 marks)

□

Question 4 (*15 marks*)**.** Let m and n be integers. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

Proof. "$\rightarrow$"(5 marks, for knowing to prove the "only if part $\rightarrow$" and the "if part $\leftarrow$" separately): Suppose $m^2 = n^2$. Then

$$m^2 - n^2 = (m + n)(m - n) = 0.$$

Thus $m + n = 0$ or $m - n = 0$, and it follows that $m = n$ or $m = -n$. (5 marks)

"$\leftarrow$": If $m = n$, then it is clear that $m^2 = n^2$. If $m = -n$, then $m^2 = (-n)^2 = (-1)^2 n^2 = n^2$. (5 marks) $\square$