Fundamentals of Mathematics (MATH 1510)

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Outline

Quadratic functions

Definition

A quadratic function is a function of the form

$$f(x) = ax^2 + bx + c,$$

where a, b, c are constants and $a \neq 0$.

Standard form of a quadratic functions

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the standard form

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with vertex (h, k); the parabola opens upward if $a > 0$ or downward if $a < 0$.

Standard form of a quadratic functions

Example

Let $f(x) = 2x^2 - 12x + 13$.

(1) Express f in standard form.

(2) Find the vertex and x- and y-intercepts of f.

(3) Find the domain and range of f.

(4) Sketch the graph of f.

Standard form of a quadratic functions

Solution.

(1) $f(x) = 2(x^2 - 6x + 9) - 5 = 2(x-3)^2 - 5$.

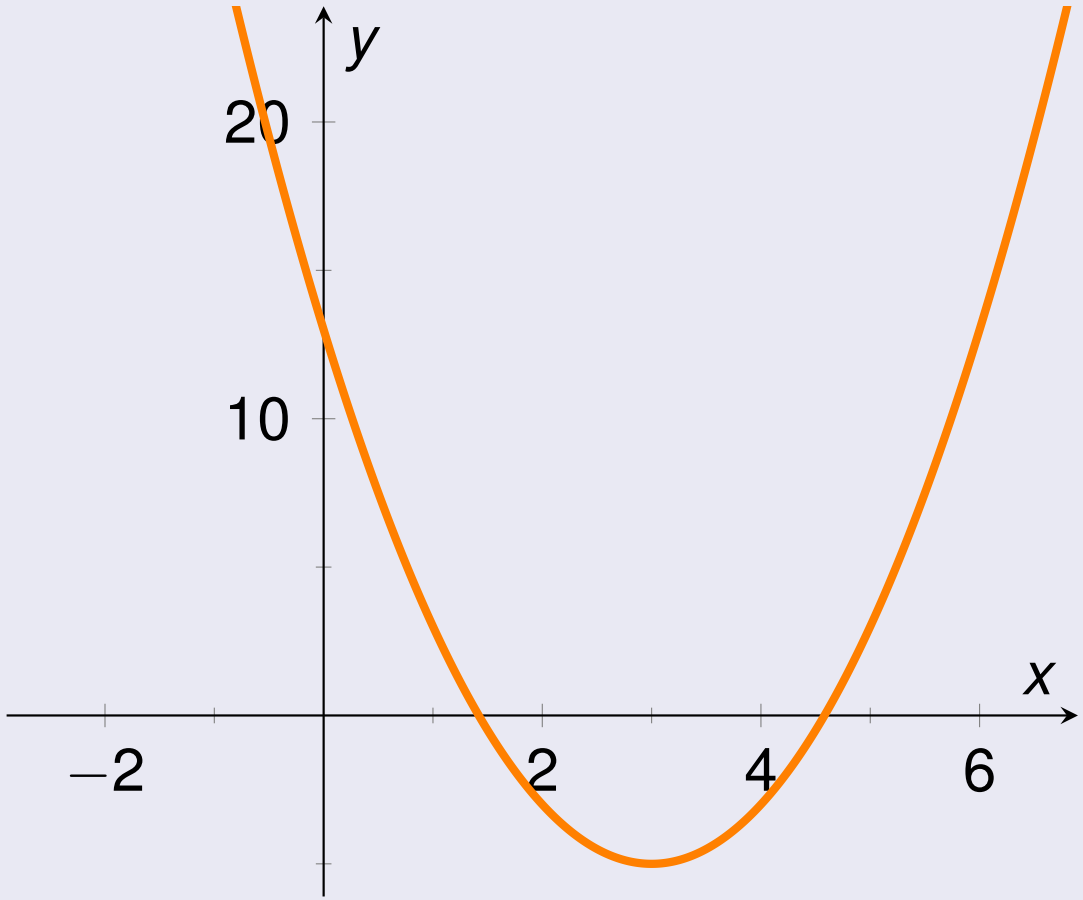
(2) The vertex is $(3, -5)$. The y-intercept of f is $f(0) = 13$, and the x-intercepts are

$$\frac{12 \pm \sqrt{144 - 4 \cdot 2 \cdot 13}}{4} = \frac{6 \pm \sqrt{10}}{2}.$$

(3) The domain of f is $\mathbb{R}$, and the range of f is $[-5, \infty)$.

Standard form of a quadratic functions

(4)



Extrema of quadratic functions

Proposition

The extremum (pl. extrema) of a quadratic function $f(x) = a(x-h)^2 + k$ occurs at $x = h$.

- *If $a > 0$, then $f(h) = k$ is the minimum (pl. minima) of f.*
- *If $a < 0$, then $f(h) = k$ is the maximum (pl. maxima) of f.*

Extrema of quadratic functions

Example

Find the extrema of the following quadratic functions:

(1) $f(x) = 5x^2 - 30x + 49$.

(2) $f(x) = -x^2 + x + 2$.

Extrema of quadratic functions

Solution.

(1) $f(x) = 5(x^2 - 6x + 9) + 4 = 5(x-3)^2 + 4$. Thus $f(3) = 4$ is the minimum of f.

(2) $f(x) = -\left(x^2 - x + \frac{1}{4}\right) + \frac{9}{4} = -\left(x - \frac{1}{2}\right)^2 + \frac{9}{4}$. Thus $f\left(\frac{1}{2}\right) = \frac{9}{4}$ is the maximum of f.

Extrema of quadratic functions

In general, a quadratic function $f(x) = ax^2 + bx + c$ may be factorized as

$$f(x) = a\Big(x + \frac{b}{2a}\Big)^2 + c - \frac{b^2}{4a}.$$

Thus $f\Big(-\frac{b}{2a}\Big) = c - \frac{b^2}{4a}$ is the extremum of f, which is a minimum when $a > 0$ and a maximum when $a < 0$.