Fundamentals of Mathematics (MATH 1510)

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Outline

Polynomial inequalities

Example

Solve the inequality

$$2x^3 + x^2 + 6 \geq 13x.$$

Polynomial inequalities

Solution.

$$2x^3 + x^2 - 13x + 6 \geq 0,$$
$$(x-2)(2x-1)(x+3) \geq 0.$$

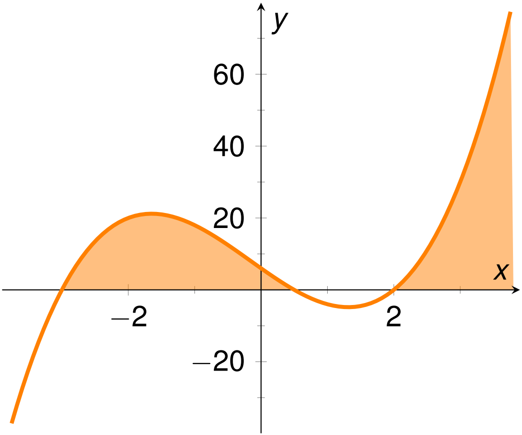
Checking the sign:

	$(-\infty, -3)$	$(-3, \frac{1}{2})$	$(\frac{1}{2}, 2)$	$(2, \infty)$
$x-2$	$-$	$-$	$-$	$+$
$2x-1$	$-$	$-$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$

By checking the endpoints, we obtain the solution set $[-3, \frac{1}{2}] \cup [2, \infty)$. □

Polynomial inequalities

The graph of $f(x) = 2x^3 + x^2 - 13x + 6$ confirms our solution:



Rational inequalities

Example

Solve the inequality

$$\frac{1-2x}{x^2-2x-3} \geq 1.$$

Rational inequalities

Solution.

$$\frac{1-2x}{x^2-2x-3}-1 \geq 0,$$

$$\frac{4-x^2}{x^2-2x-3} \geq 0.$$

$$\frac{x^2-4}{x^2-2x-3} \leq 0.$$

$$\frac{(x-2)(x+2)}{(x-3)(x+1)} \leq 0.$$

Rational inequalities

Checking the sign:

	$(-\infty, -2)$	$(-2, -1)$	$(-1, 2)$	$(2, 3)$	$(3, \infty)$
$x - 3$	$-$	$-$	$-$	$-$	$+$
$x - 2$	$-$	$-$	$-$	$+$	$+$
$x + 1$	$-$	$-$	$+$	$+$	$+$
$x + 2$	$-$	$+$	$+$	$+$	$+$

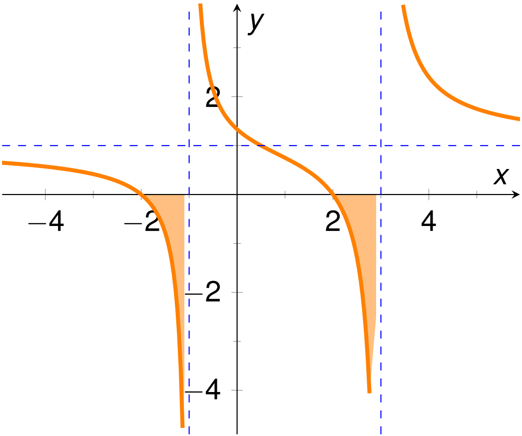
By checking the endpoints, we obtain the solution set

$$[-2, -1) \cup [2, 3).$$

□

Polynomial inequalities

The graph of $f(x) = \dfrac{x^2 - 4}{x^2 - 2x - 3}$ confirms our solution:



Outline

Systems of linear equations and solutions

A linear equation in two variables x and y is an equation in the form of

$$ax + by = c,$$

where $a, b, c \in \mathbb{R}$ are constants.

A system of linear equations in two variables consists of several linear equations that involve the same variables.

Systems of linear equations and solutions

Here is an example of a system of linear equations in two variables:

$$\begin{cases} 2x - \ \ y = 5 \\ \ \ x + 4y = 7 \end{cases}$$

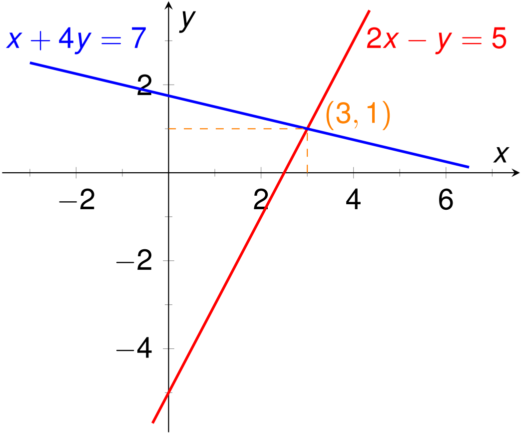
We can check that

$$\begin{cases} x = 3 \\ y = 1 \end{cases}$$

is a solution of this system.

Systems of linear equations and solutions

In fact, $(3, 1)$ is the coordinate of the intersection of the two lines:



Substitution method

Example

Find all solutions of the system

$$\begin{cases} 2x + y = 1 & (1) \\ 3x + 4y = 14 & (2) \end{cases}$$

Substitution method

Solution.

From (1) one has $y = 1 - 2x$, and it follows from (2) that

$$\begin{aligned} 3x + 4(1 - 2x) &= 14, \\ -5x &= 10, \\ x &= -2. \end{aligned}$$

Hence

$$\begin{cases} x = -2 \\ y = 1 - 2 \cdot (-2) = 5 \end{cases}$$

is the solution of the system. □

Elimination method

Example

Find all solutions of the system

$$\begin{cases} 3x + 2y = 14 & (1) \\ x - 2y = 2 & (2) \end{cases}$$

Elimination method

Solution.

By (1)+(2) one has

$$4x = 16$$

and thus $x = 4$. Hence

$$4 - 2y = 2$$

and consequently $y = 1$. Therefore,

$$\begin{cases} x = 4 \\ y = 1 \end{cases}$$

is the solution of the system. □

The number of solutions

For a system of linear equations in two variables, exactly one of the following is true:

1. The system has exactly one solution.
2. The system has no solution.
3. The system has infinitely many solutions.

A linear system with one solution

Example

Solve the system

$$\begin{cases} 3x - y = 0 & \color{blue}{(1)} \\ 5x + 2y = 22 & \color{blue}{(2)} \end{cases}$$

A linear system with one solution

Solution.

By $2 \cdot (1) + (2)$ one has

$$11x = 22$$

and thus $x = 2$. It follows that

$$y = 3 \cdot 2 = 6.$$

Therefore,

$$\begin{cases} x = 2 \\ y = 6 \end{cases}$$

is the solution of the system. □

A linear system with no solution

Example

Solve the system

$$\begin{cases} 8x - 2y = 5 & (1) \\ -12x + 3y = 7 & (2) \end{cases}$$

A linear system with no solution

Solution.

By $3 \cdot (1) + 2 \cdot (2)$ one has

$$0 = 29.$$

Thus the system has no solution. □

A linear system with infinitely many solutions

Example

Solve the system

$$\begin{cases} 3x - 6y = 12 & (1) \\ 4x - 8y = 16 & (2) \end{cases}$$

A linear system with infinitely many solutions

Solution.

Equations (1) and (2) can be both simplified to

$$x - 2y = 4.$$

Thus the system has infinitely many solutions in the form of

$$\begin{cases} x = t \\ y = \frac{1}{2}t - 2 \end{cases}$$

where $t \in \mathbb{R}$. □

Modeling with linear equations

Example

There are some chickens and rabbits staying in the same cage. If there are 35 heads and 94 legs, how many chickens and rabbits are there?

Modeling with linear equations

Solution.

Suppose there are x chickens and y rabbits. Then

$$\begin{cases} x + y = 35 & (1) \\ 2x + 4y = 94 & (2) \end{cases}$$

By (2) $- 2 \cdot$ (1) one has $2y = 24$ and consequently $y = 12$. It follows that

$$\begin{cases} x = 35 - 12 = 23 \\ y = 12 \end{cases}$$

is the solution. Therefore, there are 23 chickens and 12 rabbits in the cage. □