Fundamentals of Mathematics (MATH 1510)

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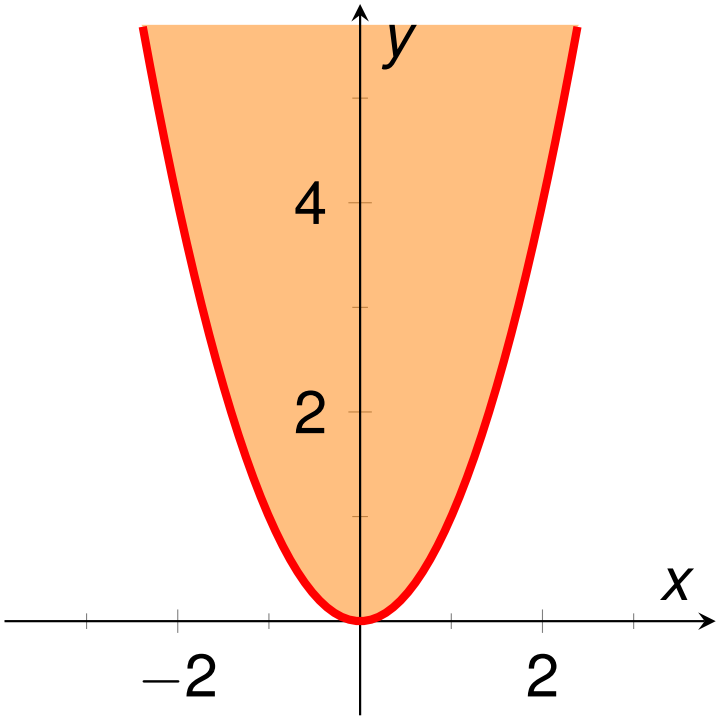
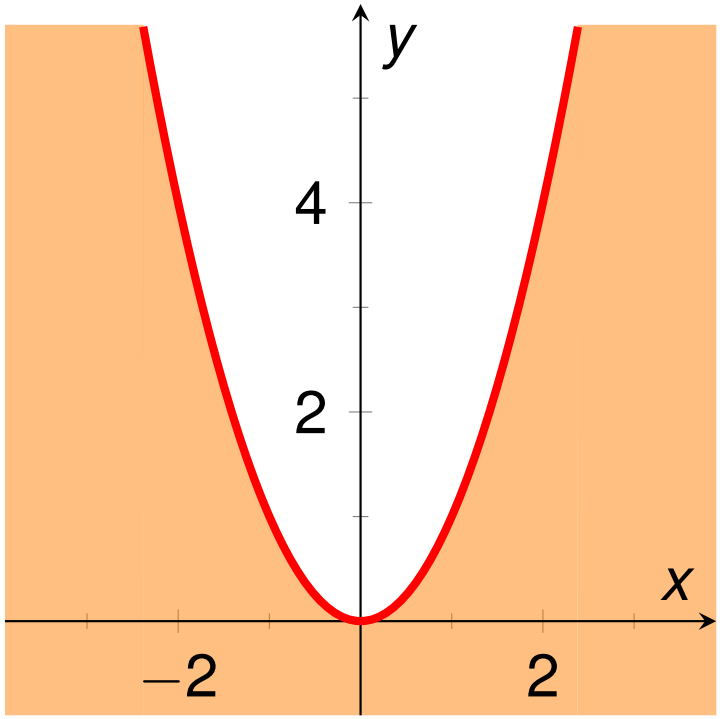
Outline

Graphing an inequality

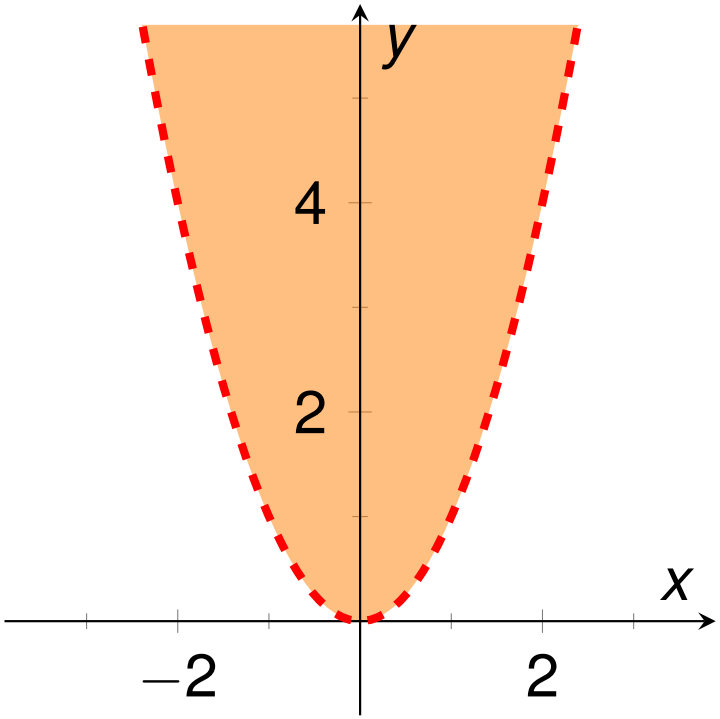


$y \geq x^2$

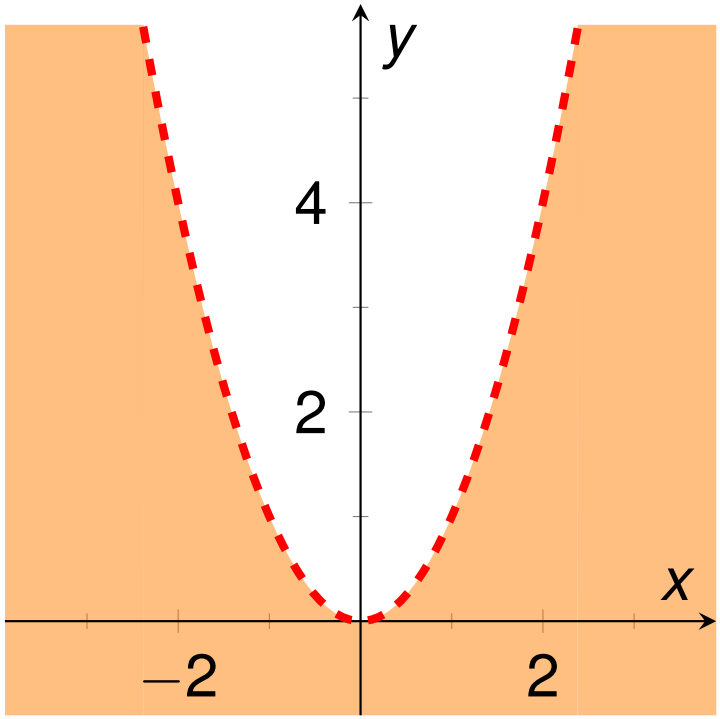


$y \leq x^2$

Graphing an inequality



$y > x^2$



$y < x^2$

Graphs of inequalities

Example

Graph the following inequalities:

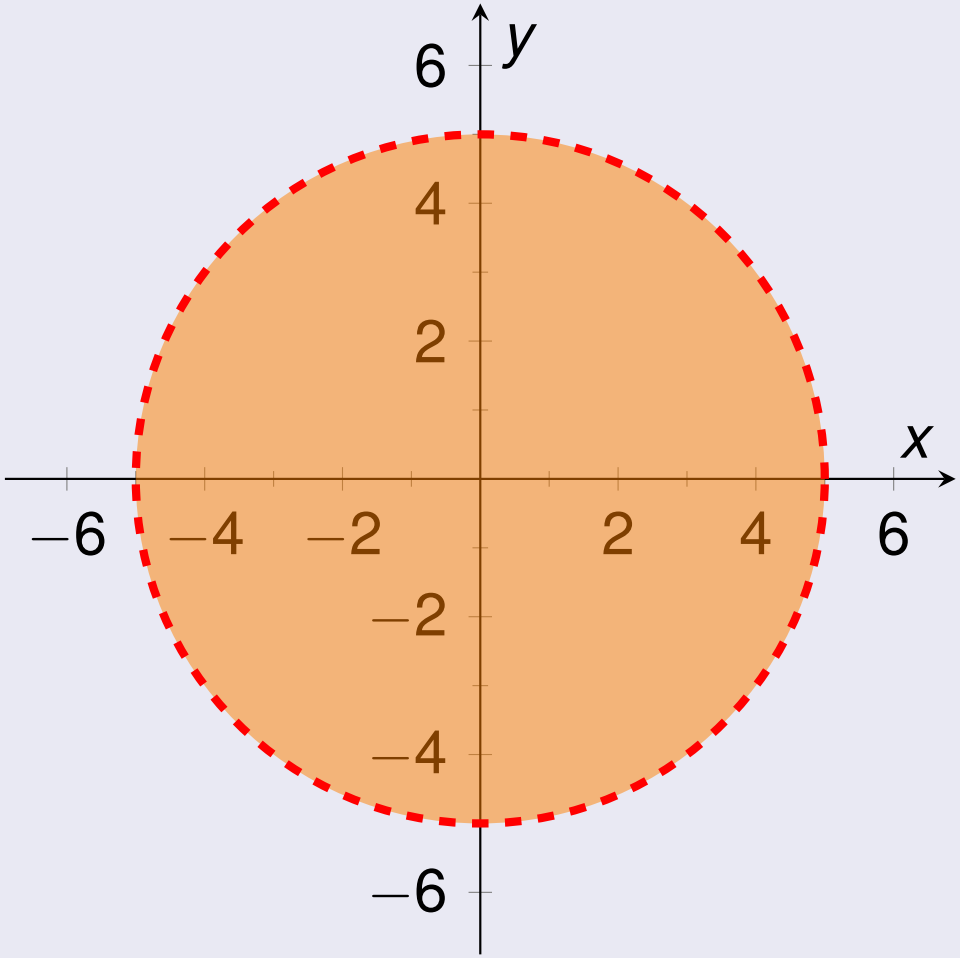
(1) $x^2 + y^2 < 25$.

(2) $x + 2y \geq 5$.

Graphs of inequalities

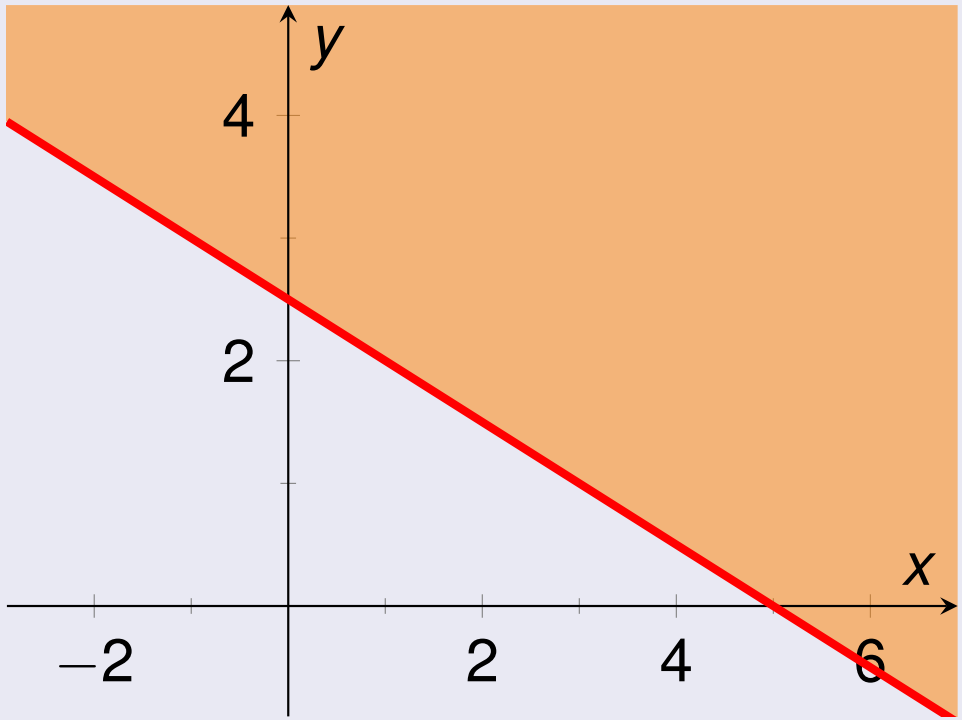
Solution.

(1) $x^2 + y^2 < 25$:



Graphs of inequalities

(2) $x + 2y \geq 5$:



Systems of inequalities

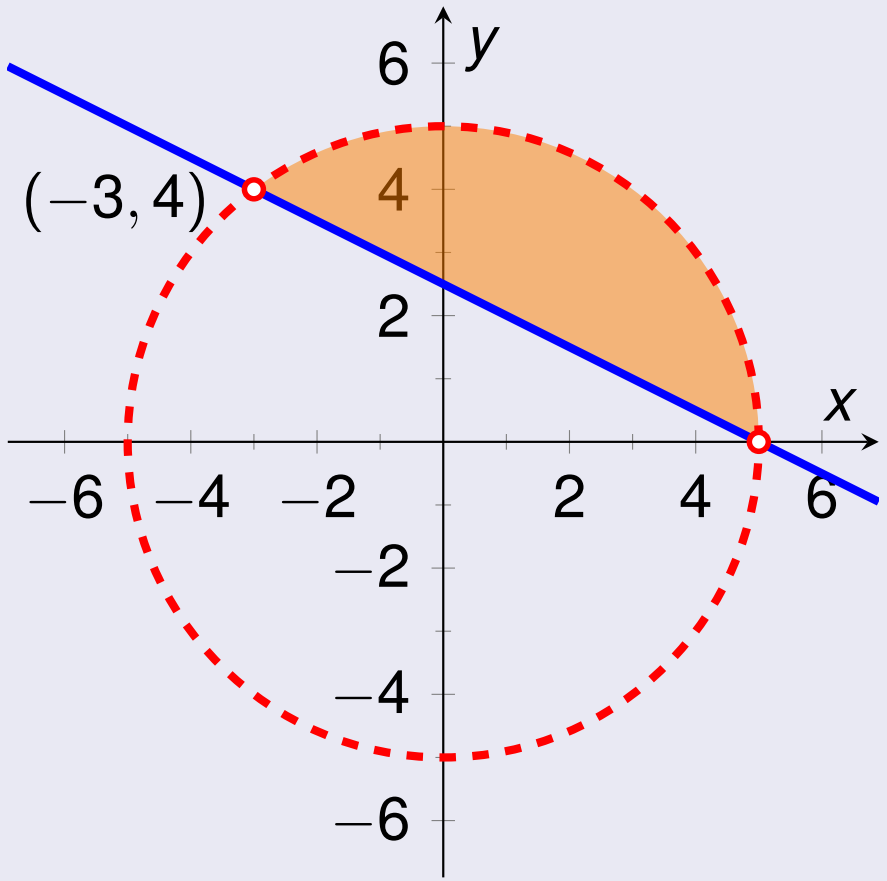
Example

Graph the solution set of the system of inequalities

$$\begin{cases} x^2 + y^2 < 25 \\ x + 2y \geq 5 \end{cases}$$

Systems of inequalities

Solution.



Systems of linear inequalities

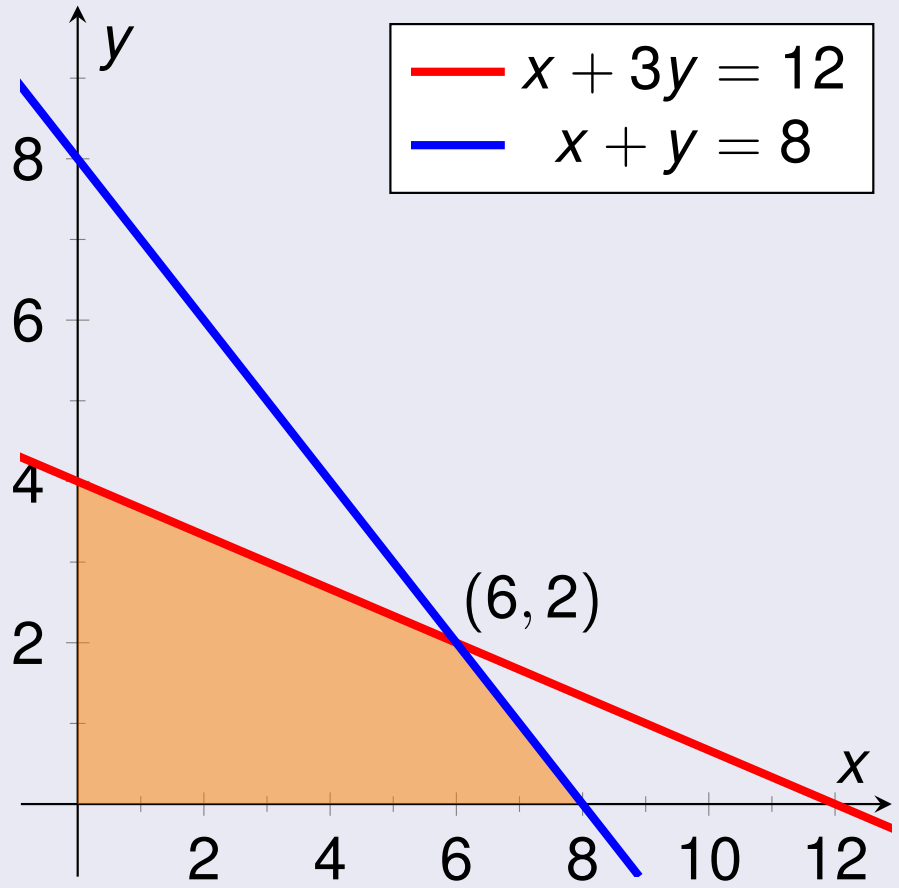
Example

Graph the solution set of the system of linear inequalities

$$\begin{cases} x + 3y \leq 12 \\ x + y \leq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Systems of linear inequalities

Solution.



Systems of linear inequalities

Example

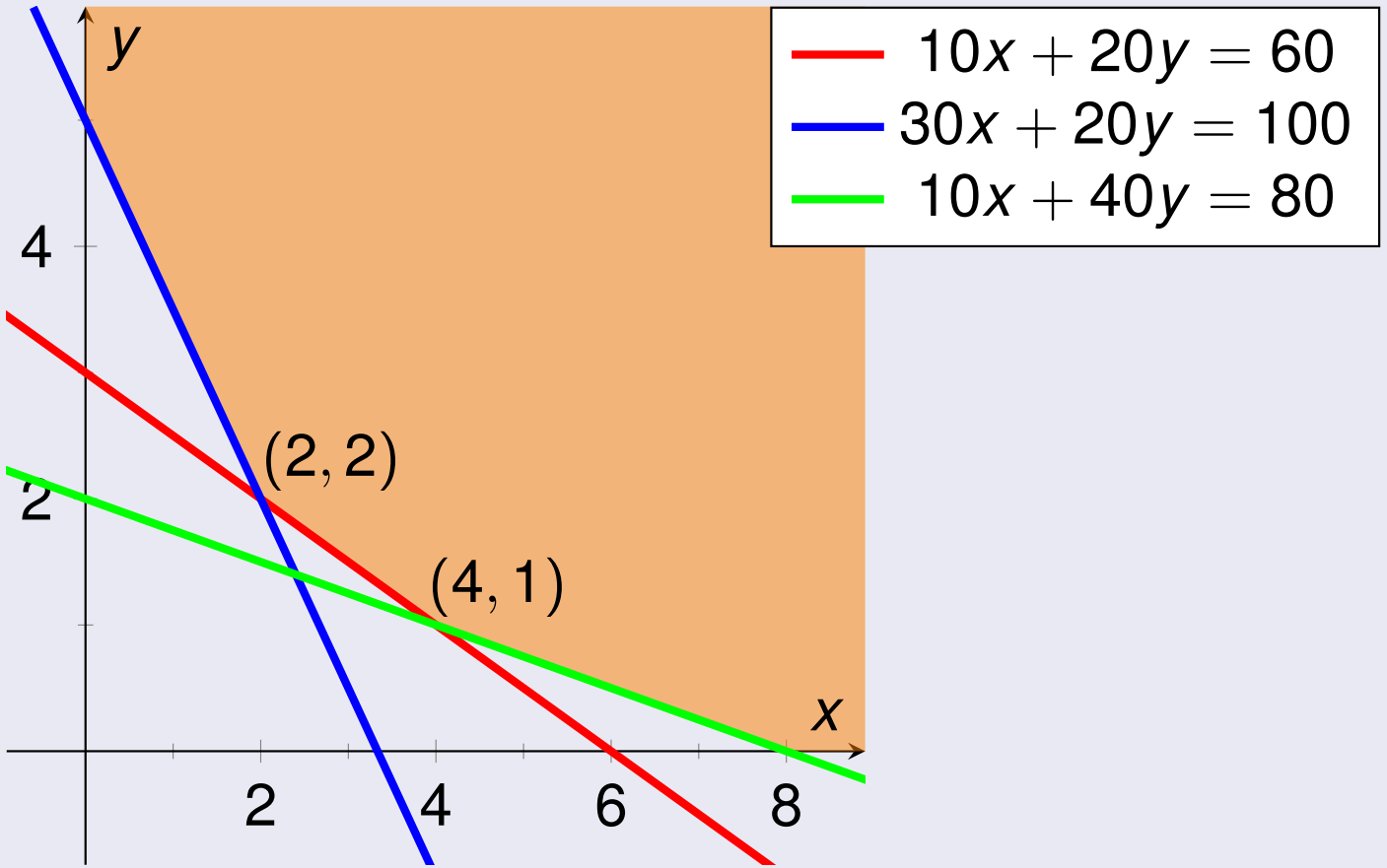
Graph the solution set of the system of linear inequalities

$$(1)\begin{cases} 10x + 20y \geq 60 \\ 30x + 20y \geq 100 \\ 10x + 40y \geq 80 \\ x \geq 0 \\ y \geq 0 \end{cases} \qquad (2)\begin{cases} 10x + 20y \leq 60 \\ 30x + 20y \geq 100 \\ 10x + 40y \geq 80 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Systems of linear inequalities

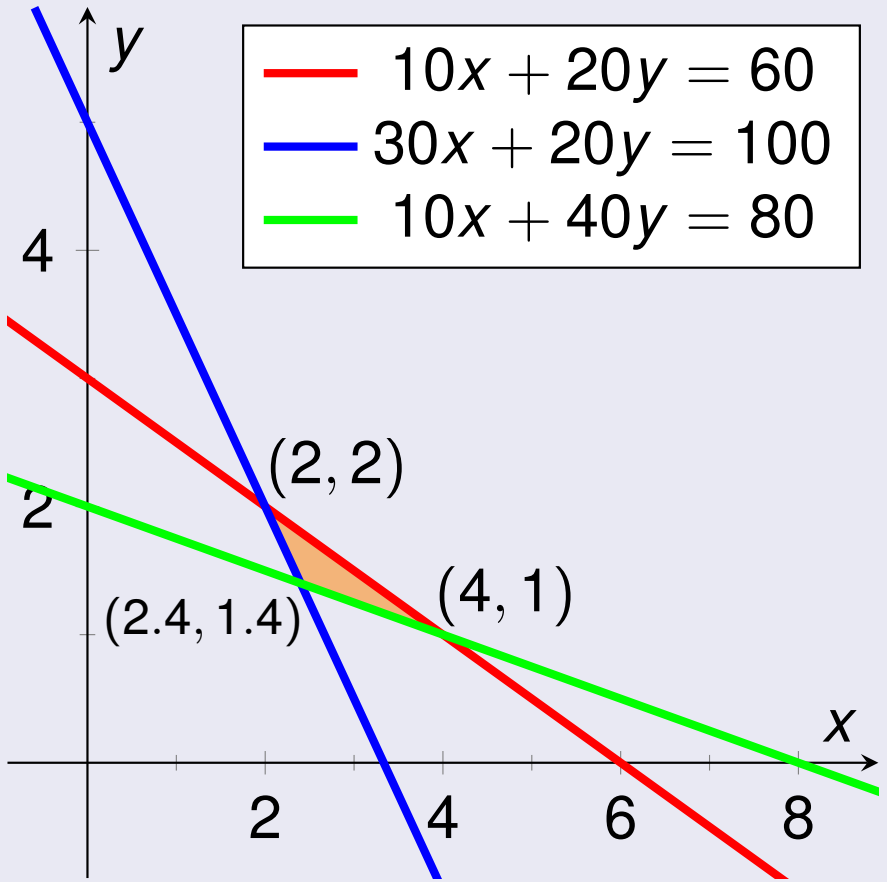
Solution.

(1)



Systems of linear inequalities

(2)



Bounded and unbounded regions

A region in the plane is called bounded if it can be enclosed in a (sufficiently large) circle. A region that is not bounded is called unbounded.

In the last example, the solution set of the system (1) is an unbounded region, while the solution set of the system (2) is a bounded region.

Feasible regions

Many applied problems involve constraints on the variables.

When dealing with applied inequalities, we usually refer to the solution set of a system as a feasible region, because the points in the solution set represent feasible (or possible) values for the quantities being studied.

Feasible regions

Example

A factory produces two agricultural pesticides, A and B.

- For every barrel of pesticide A, the factory emits 0.25 kg of carbon monoxide (CO) and 0.60 kg of sulfur dioxide (SO_2).
- For every barrel of pesticide B, the factory emits 0.50 kg of CO and 0.20 kg of SO_2.

Pollution laws restrict the factory's output of CO to a maximum of 75 kg per day and its output of SO_2 to a maximum of 90 kg per day.

Feasible regions

(1) Find a system of inequalities that describes the number of barrels of each pesticide the factory can produce per day and still satisfy the pollution laws. Graph the feasible region.

(2) Would it be legal for the factory to produce 100 barrels of pesticide A and 80 barrels of pesticide B per day?

(3) Would it be legal for the factory to produce 60 barrels of pesticide A and 160 barrels of pesticide B per day?

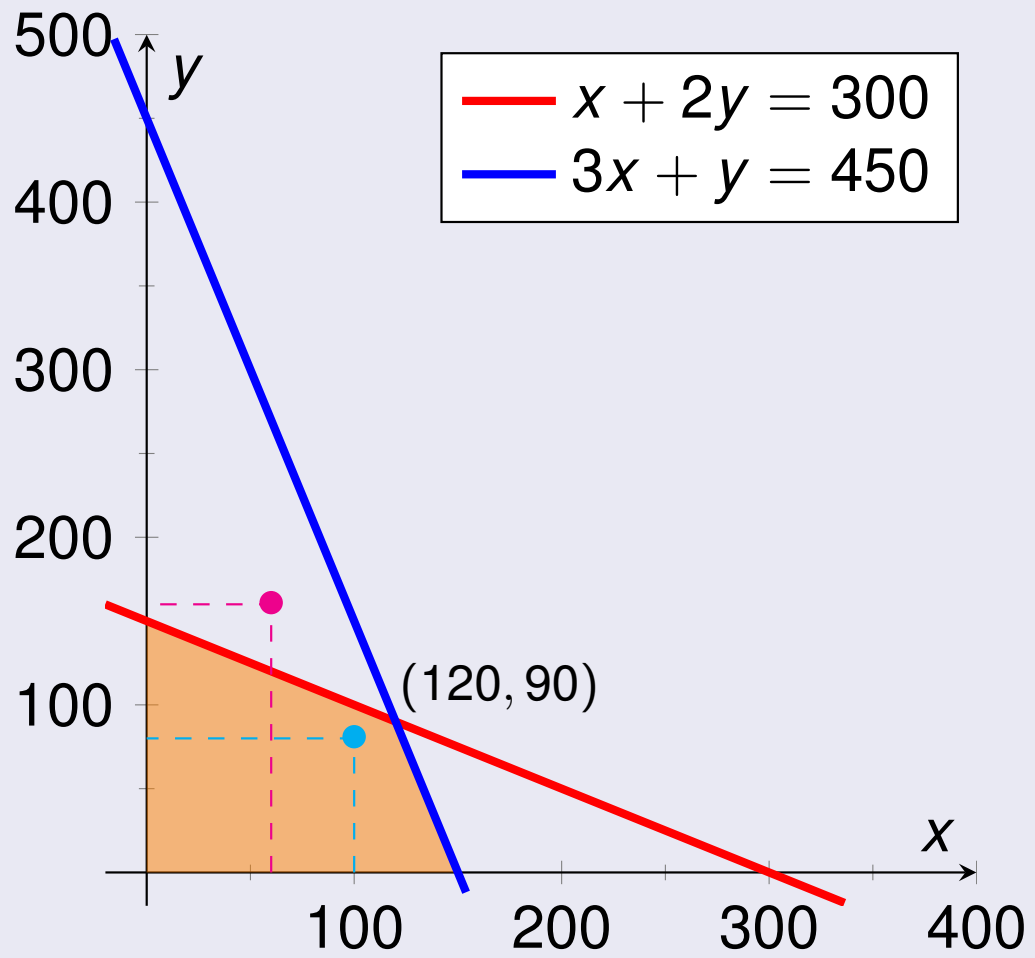
Feasible regions

Solution.

(1) Let x and y denote the number of barrels of A and B produced per day, respectively. Then

$$\begin{cases} 0.25x + 0.5y \leq 75 \\ 0.6x + 0.2y \leq 90 \\ x \geq 0 \\ y \geq 0 \end{cases} \Longrightarrow \begin{cases} x + 2y \leq 300 \\ 3x + y \leq 450 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Feasible regions



Feasible regions

(2) Since the point $(100, 80)$ lies inside the feasible region (see the cyan dot in the graph), this production plan is legal.

(3) Since the point $(60, 160)$ lies outside the feasible region, this production plan is not legal (see the magenta dot in the graph). It violates the CO restriction, although it does not violate the SO_2 restriction.

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